June 2018

Investigation into the validity of using the Glasstone & Dolan formula for the modelling of tsunami waves generated by an asteroid impact

by D J Gardner

Introduction

The purpose of this study is to investigate whether an asteroid impact into the ocean can be modelled using the simple approach of regarding it as being similar to an explosion caused by an underwater nuclear weapon, for which a formula for the surface wave height generated at a specified radial distance from the explosion is available in [1]. An asteroid impact into the ocean could potentially involve an amount of explosive energy that is orders of magnitude higher than a nuclear weapon, so there is some uncertainty as to whether the formula would still be valid in this situation. To check whether application of the formula is valid, it is proposed to identify a suitable asteroid impact test problem from the tsunami modelling literature, and then model the tsunami waves that are generated using the incompressible Navier-Stokes program SOLA-3D. Coding for the SOLA-3D program is supplied with the book [2], and the program has been used previously for work on tsunami waves generated by an underwater landslide as reported in [3].

Glasstone & Dolan formula

The waveform for the surface waves produced by an underwater explosion has the form of a 'wave train' or 'wave packet' according to [1] as shown in Figure 1:

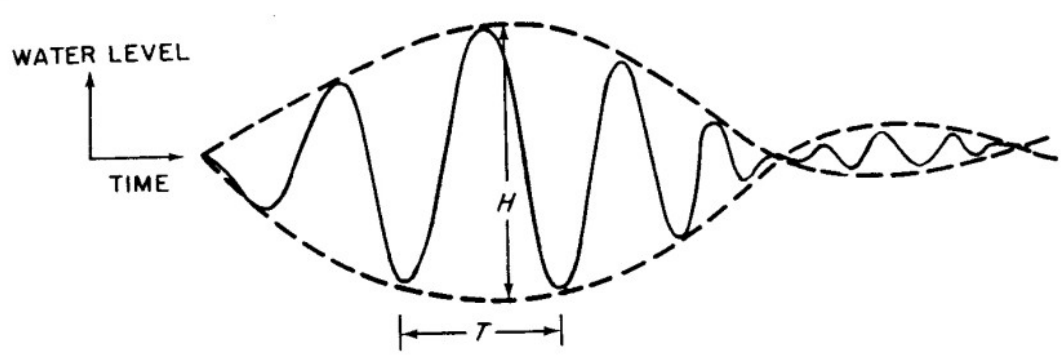


Figure 1 Waveform of surface waves produced by underwater explosion according to Glasstone & Dolan

For an explosion where the depth of water is sufficient to contain the blast (with no crater formed in the seabed), the wave height H at a radial distance R is given by:

$$H = 40500 \frac{W^{0.54}}{R}$$ where H and R are in feet, W is in kT or kilotons TNT equivalent

The depth of water d_w in which the surface waves are produced is required to be in the range 256 $W^{0.25}$ to 850 $W^{0.25}$, where d_w is in feet.

If H, R and d_w are all in metre units, the formula becomes:

$$H = 3762.66 \frac{W^{0.54}}{R}$$ with d_w required to be in the range 78.03 $W^{0.25}$ to 259.08 $W^{0.25}$

The accuracy for H is given as being about ±35 %.

An estimate for the period T in Figure 1 is also given in [1]. The estimate is based on data for underwater chemical explosions rather than nuclear explosions:

$$T = 14.1\ W^{0.144}$$ where W is in kT or kilotons TNT equivalent

For an asteroid impact into the ocean, W would be determined by converting the impact kinetic energy of the asteroid in Joules to kT units using the relation 1 kT = 4.184 x 10^{12} Joules.

The formula for H in [1] is understood to be based on two underwater nuclear weapons tests carried out by the USA in the 1950s. The first test was a 32 kT weapon exploded in May 1955 at a depth of 2000 feet in 16,000 feet deep water. The second test was a 9 kT weapon exploded in May 1958 at a depth of 500 feet in 3000 feet deep water. Further details of these tests are given in [4].

No information is provided in [1] regarding the number of cycles in the wave train or the shape of the wave train envelope (the broken lines in Figure 1). So assumptions may have to be made about these parameters in applying the formula to determine an input wave motion for a program that is used to model the propagation of tsunami waves from the vicinity of the asteroid impact site to a coastline.

Asteroid impact test problem

A suitable asteroid impact test problem is available in [5], which gives results for the maximum cavity size generated in a 5000 m deep ocean by a spherical asteroid travelling vertically downwards. The asteroid is assumed to hit the ocean surface at a speed of 20 km/sec, and is composed of a stony material of density 3.32 g/cm^3. Diameters of 250 m, 500 m and 1000 m are assumed for the asteroid. The asteroid explosion in the ocean is modelled in [5] using a program called CTH. The maximum cavity size is then used as an initial input condition for the incompressible Navier-Stokes program ZUNI, which models the subsequent filling of the cavity and calculates peak surface wave elevations at distances up to 150 km from the centre of the impact site.

An extended version of this test problem is considered in [6], which looks at asteroids of iron composition as well as the stony material. The stony material with the density of 3.32 g/cm^3 is identified as being dunite. A compressible Navier-Stokes program called SAGE is used to model both the asteroid explosion in the ocean and the subsequent propagation of surface waves at some distance from the explosion. Maximum cavity sizes that are generated for the various asteroid explosion cases are also provided in [6].

The case selected for analysis on SOLA-3D is shown in Figures 2a and 2b taken from [5]. This case corresponds to a 500 m diameter asteroid of dunite material. From Figure 2b, the maximum cavity size is estimated as having a radius of about 5000 m, and a maximum depth of about 4500 m. The cavity appears to have a semi-ellipsoid shape. Figure 2b also indicates that cratering of the seabed has not occurred, and the seabed appears to be only slightly deformed by the explosion.

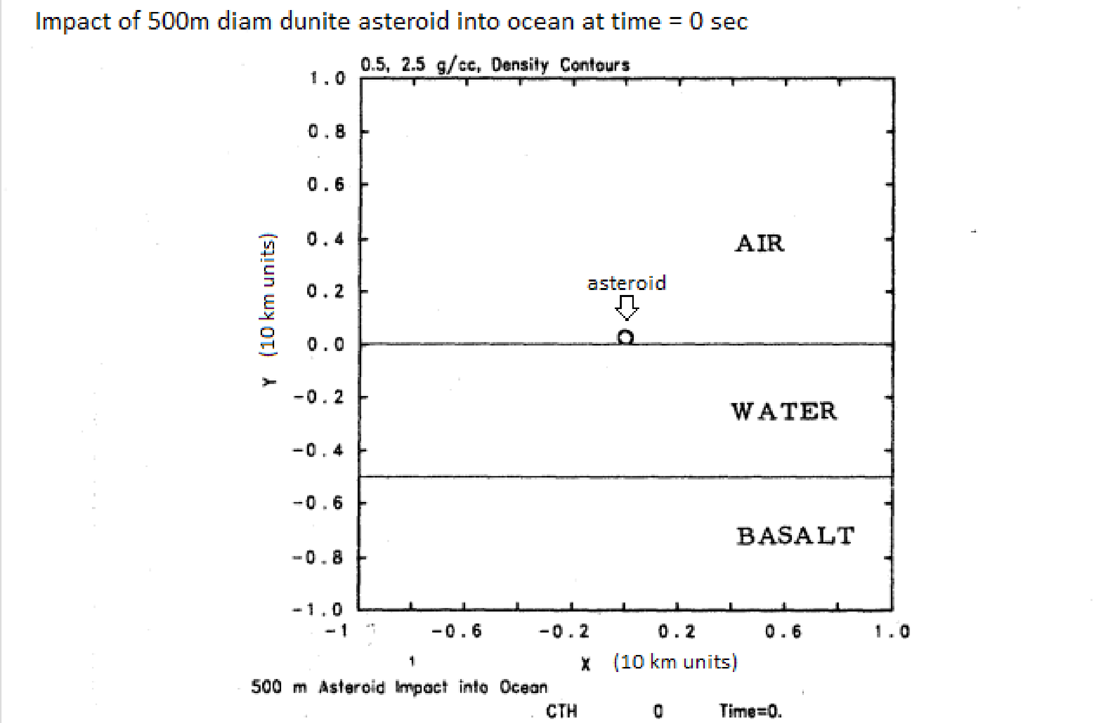


Figure 2a

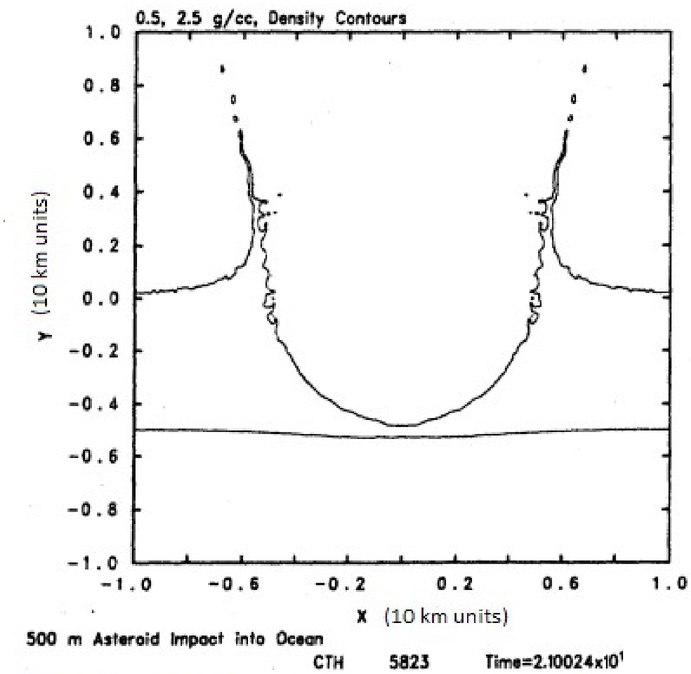


Figure 2b Impact of 500m diam dunite asteroid into ocean at time = 21 sec

The kinetic energy of the 500 m diameter dunite asteroid travelling at 20 km/sec is represented in the Glasstone & Dolan formula by a W value of 10,386,878.2 kT or about 10,400 MT (megatons TNT equivalent). For this W value, d_w is required to be in the range 4429.8 m to 14708.1 m, which is satisfied for an ocean depth of 5000 m. The estimate for the period T for this W value is 144 seconds.

A W value of 10400 MT represents a very large extrapolation of the Glasstone & Dolan formula from the test data that it is based on, which only goes up to 32 kT. A W value of 32 kT would correspond to a comparatively much smaller stony asteroid of 7.28 m diameter travelling at 20 km/sec, and at this size, the asteroid would probably be described as a meteor or a meteorite.

SOLA-3D analysis of test problem

The selected asteroid impact test problem is modelled on SOLA-3D using a quarter model with the computational grid depicted in Figure 3. The 5000 m deep ocean corresponds to the blue region in the figure.

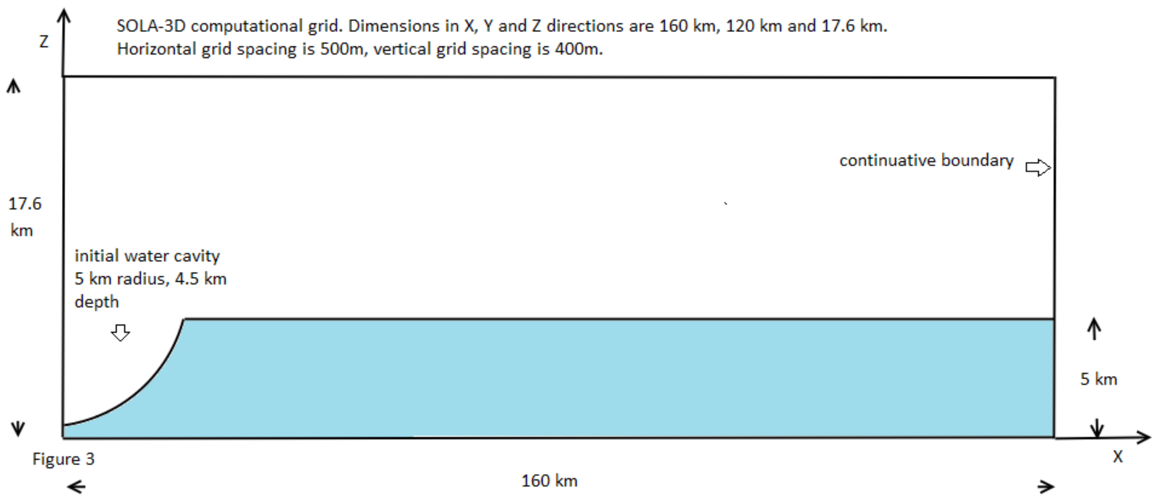


Figure 3

The computational grid dimensions are 160 km in the X direction, 120 km in the Y direction and 17.6 km in the vertical Z direction. The initial cavity caused by the explosion in the ocean is modelled as a quarter of a semi-ellipsoid volume. Surface wave time history responses are monitored near to the X axis at up to 100 km from the centre of the initial cavity (the response locations need to be kept at a fairly significant distance from the continuative boundary to avoid spurious effects on the response). The 17.6 km vertical dimension of the computational grid is required to accommodate the central vertical jet that is formed when the initial cavity is filled, and several attempts may be required to determine a suitable vertical dimension. The grid spacing is 500 m in the two horizontal directions and 400 m in the vertical direction.

The analysis was run for a duration of 1000 seconds using a constant 0.5 second time step. It would have been preferred to use automatic time stepping, but this could not be adopted as the run is unable to get past about 100 seconds due to progressively decreasing very small time steps being selected. However the response results obtained with automatic time stepping are reasonably similar to the results obtained with a constant time step of 0.5 seconds up to 100 seconds.

The number of computational grid cells is 3,379,200 and the runtime for an analysis duration of 1000 seconds was 2.8 hours on a 2.4 GHz personal computer.

Surface elevation time history results produced by SOLA-3D at distances 0.25 km, 4.75 km, 9.75 km, 19.75 km, 49.75 km, 79.75 km and 99.75 km from the centre of the initial cavity are shown in Figures 4a to 4g.

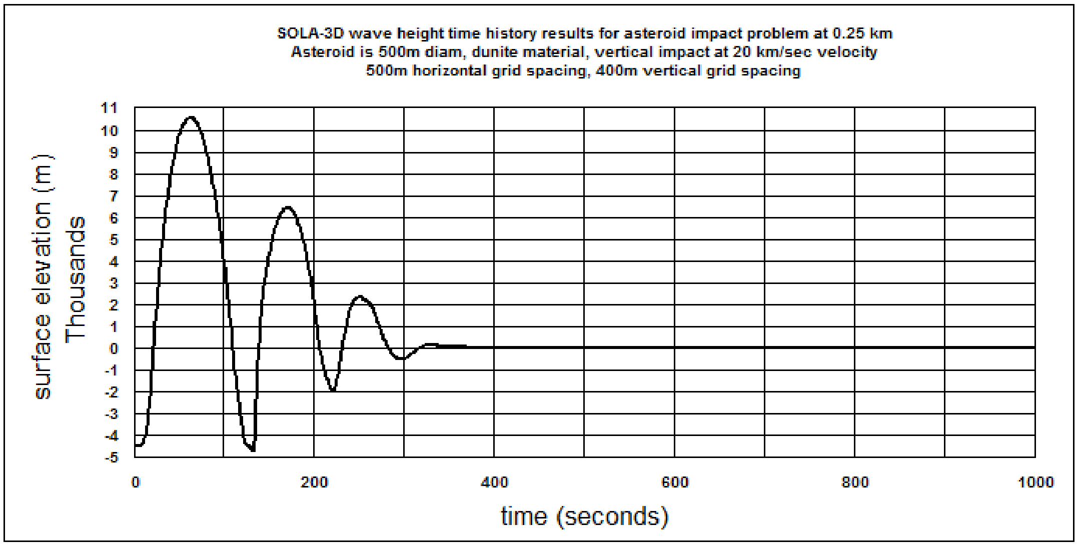


Figure 4a

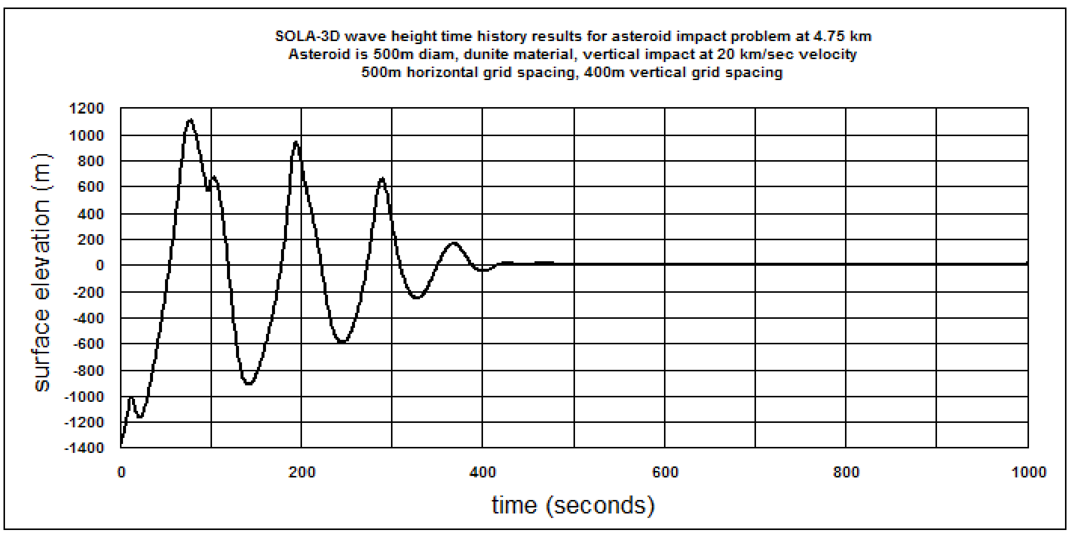


Figure 4b

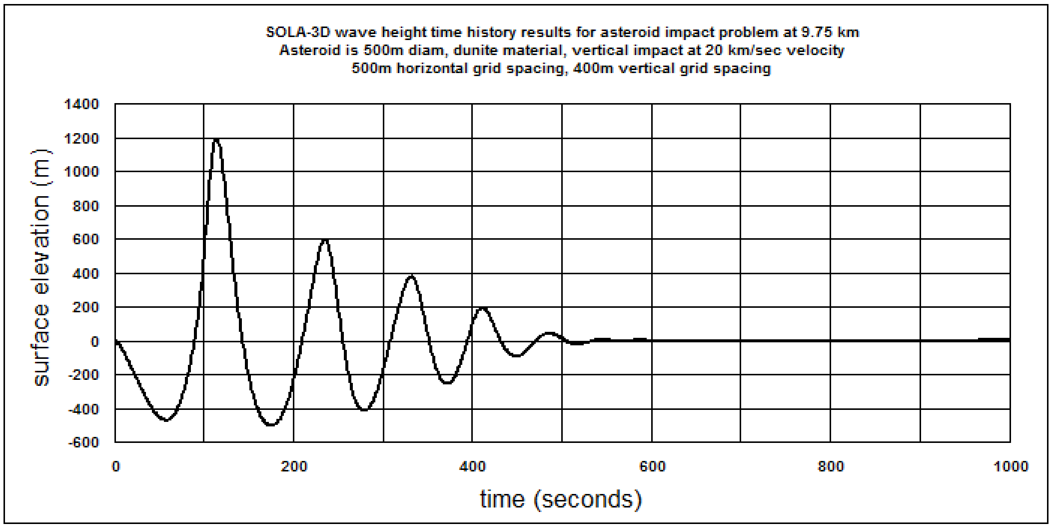


Figure 4c

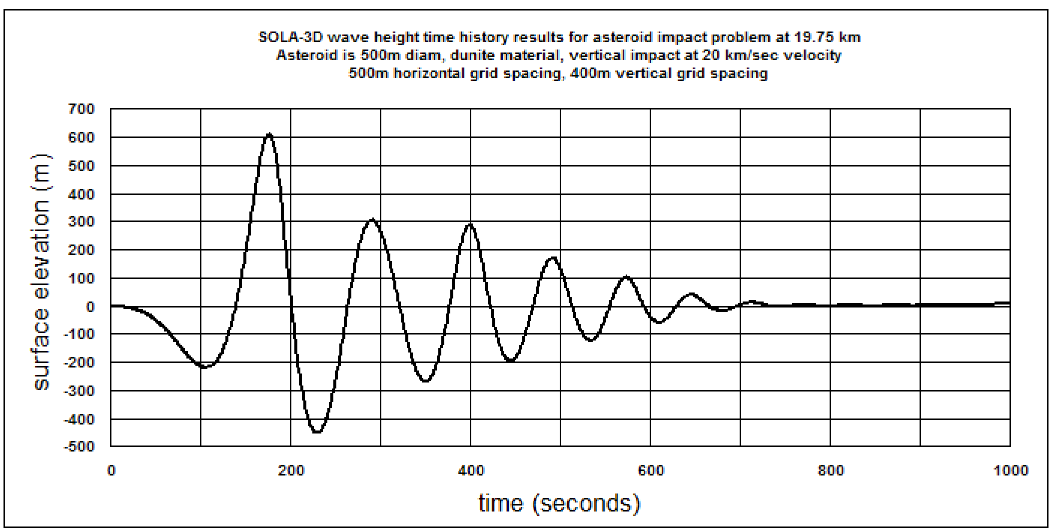


Figure 4d

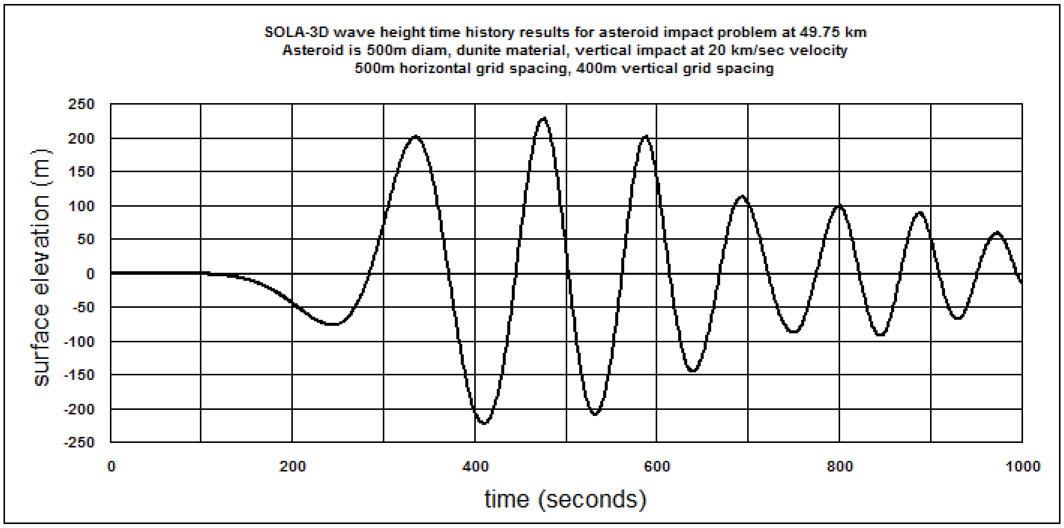


Figure 4e

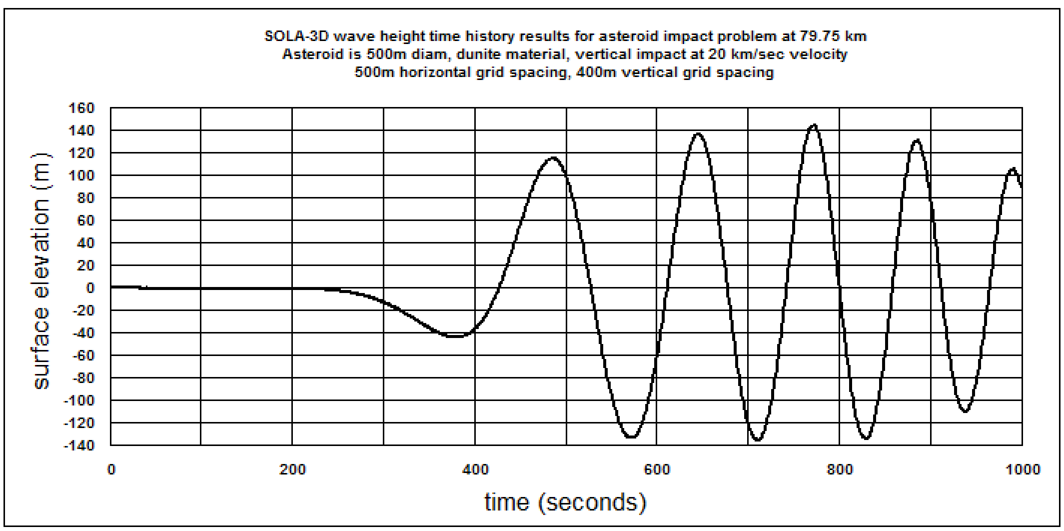


Figure 4f

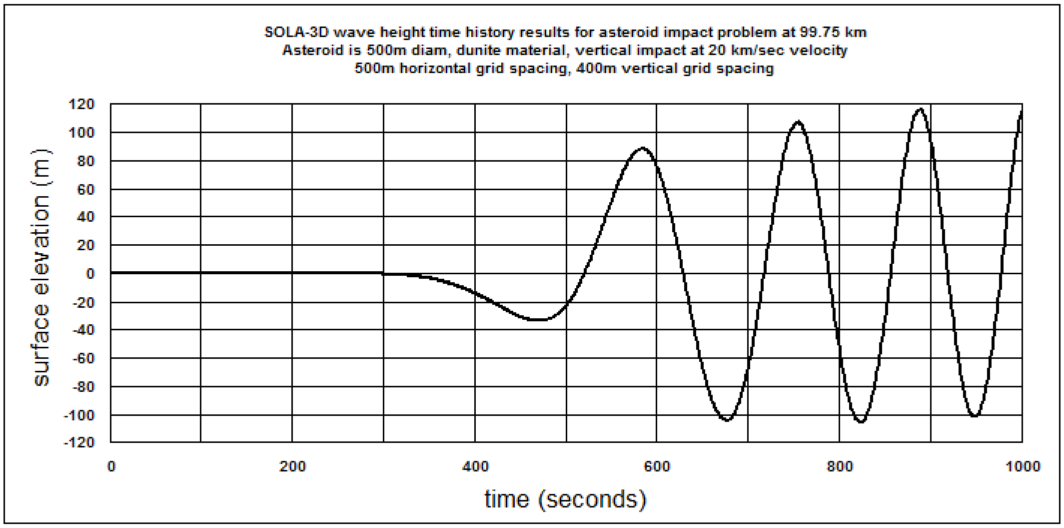


Figure 4g

The surface elevation time history results look generally similar to the waveform depicted in Figure 1, and the waveform is maintained for a distance of at least 100 km from the centre of the impact site. The wave train envelope in the SOLA-3D results shows some variability at different locations. It was thought best to interpret H as being the surface elevation range in the SOLA-3D results for comparison with the Glasstone & Dolan formula, rather than try to estimate H from the wave train envelope as it appears to be defined in Figure 1.

Comparison of SOLA-3D results with Glasstone & Dolan formula

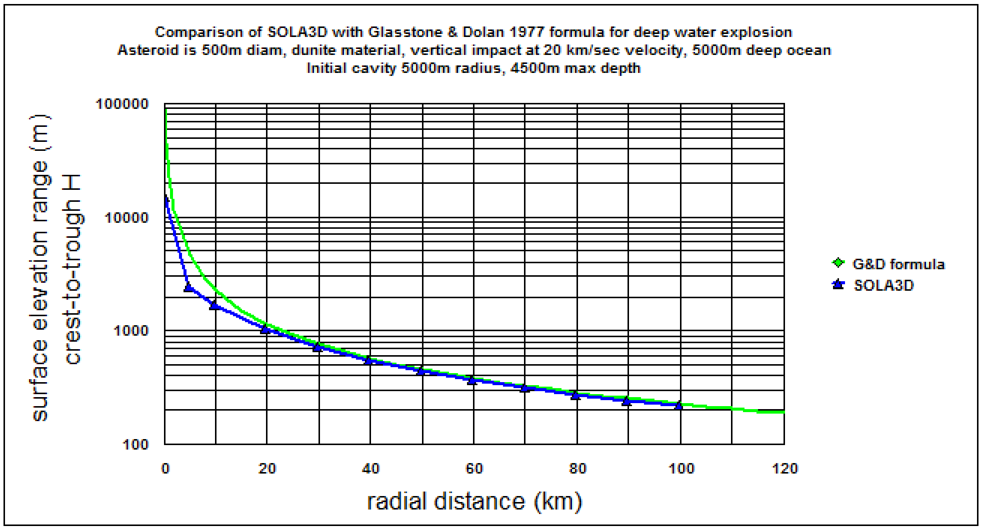


Figure 5

The surface elevation range determined from the SOLA-3D analysis shows good agreement with the Glasstone & Dolan formula for H versus radial distance for distances in the range 20 km to 100 km for this asteroid explosion case, with 100 km being the maximum distance that was considered in the SOLA-3D analysis. At distances less than 20 km, the Glasstone & Dolan formula for H is conservative relative to the SOLA-3D results.

The period T was estimated from successive troughs in the SOLA-3D time history results in Figures 4c to 4g as being in the range 117 to 125 seconds. The Glasstone & Dolan formula for T gives 144 seconds, which is 15% to 23% higher than the range of values obtained from SOLA-3D.

This comparison indicates that it is acceptable to carry out the very large extrapolation of the Glasstone & Dolan formula to cover the event of an asteroid impact into the ocean.

Comparison of SOLA-3D results with ZUNI and SAGE

Peak surface elevation results for the asteroid impact test problem are available for ZUNI in [5] and SAGE in [6], and these are compared with SOLA-3D results for the test problem in Figure 6. The SAGE results were digitised from the 'Dn 500 tr' data points in Figure 7 taken from [6]. The ZUNI and SAGE analyses use 2D axisymmetric models, whereas SOLA-3D carries out a 3D analysis, but the problem can be regarded as axisymmetric as it assumes the asteroid is travelling in a vertical direction downwards into the ocean, and the filling of the initial cavity would be expected to take place in an axisymmetric manner.

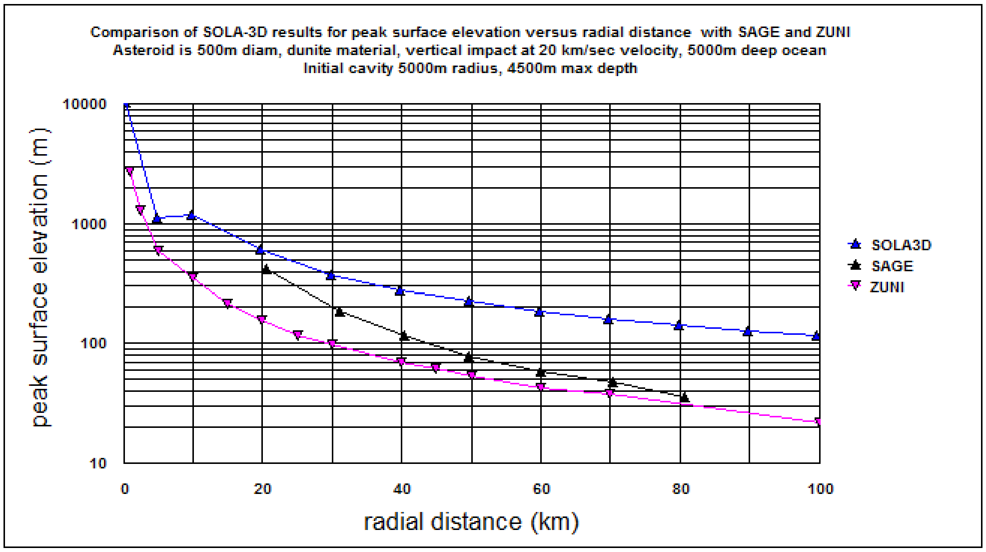
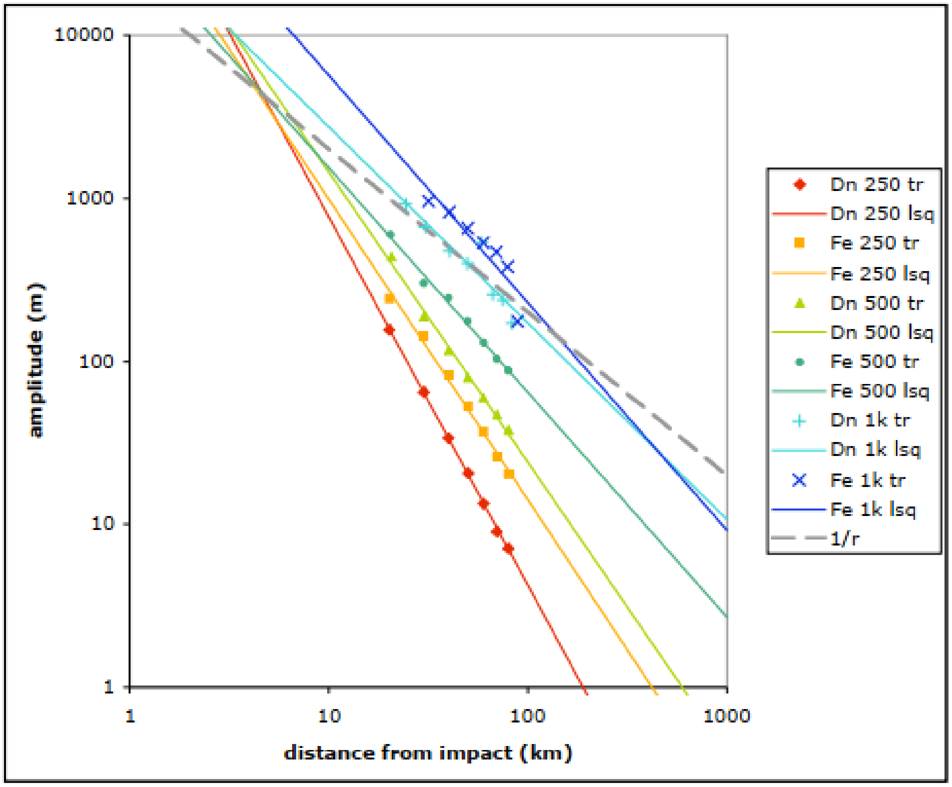

Comparison of SOLA-3D results for peak surface elevation versus radial distance with SAGE and ZUNI
Asteroid is 500m diam, dunite material, vertical impact at 20 km/sec velocity, 5000m deep ocean
Initial cavity 5000m radius, 4500m max depth
peak surface elevation (m)
10000
1000
100
10
0
20
40
60
80
100
radial distance (km)
SOLA3D
SAGE
ZUNI

Figure 6

Figure 7 SAGE results for various asteroid impact scenarios. Dn is Dunite, Fe is iron, 250, 500 and 1K refer to asteroid diameter in m, tr is actual result, lsq is least squares power law fit to results



The ZUNI and SAGE peak surface elevation results are substantially lower than the SOLA-3D results. Results from these two programs would suggest that the Glasstone & Dolan formula approach is excessively conservative for representing tsunami waves generated by asteroid impacts into the ocean.

ZUNI and SOLA-3D are both incompressible Navier-Stokes programs, and would therefore be expected to show good agreement. Coding for the ZUNI program is supplied with the book [2]. From experience in testing the ZUNI and SOLA-3D programs, ZUNI and SOLA-3D show good agreement for planar 2D problems (like the problem in Section 5A of [2]), but an initial cavity problem (such as Section 3E of [2] and [5]) could not be run on ZUNI for more than a few tenths of a second before the run failed using the version of the ZUNI coding that is supplied with [2].

A noteworthy feature of SAGE, which is a compressible Navier-Stokes program, is that it calculates a peak surface elevation attenuation rate that is significantly faster than R^{-1}, where R is the distance from the centre of the impact site. Figure 7 indicates that the attenuation rate also varies significantly for different asteroid impact scenarios. The attenuation rate for a 500 m diameter dunite asteroid is estimated from Figure 7 to be

$R^{-1.77}$, and for a smaller 250 m diameter dunite asteroid, the attenuation rate is estimated as $R^{-2.26}$.

Power law fits were also made to the SOLA-3D and ZUNI peak surface elevation data plotted in Figure 6 for distances in the range 20 to 100 km, and this gave an $R^{-1.05}$ attenuation rate for SOLA-3D and an $R^{-1.18}$ attenuation rate for ZUNI.

Compressible Navier-Stokes programs (like SAGE and CTH) are often described as 'hydrocodes' in the literature, and incompressible Navier-Stokes programs (like SOLA-3D and ZUNI) are sometimes described as 'incompressible hydrocodes'.

The book [2] strongly endorses the idea of using compressible Navier-Stokes programs for certain types of tsunami modelling, and states in the introduction: "The generation of water waves by volcanic explosions, conventional and nuclear explosions, projectiles and asteroid impacts require the use of the compressible Navier-Stokes model". However compressible Navier-Stokes programs are not supplied with the book, probably because such programs tend to have a significant monetary value, and access to this type of program tends to be restricted because of the potential for the programs to be used for military applications.

Conclusions

This study, carried out using the SOLA-3D program, provides supporting evidence that the Glasstone & Dolan formula, originally developed for prediction of the surface wave height generated by an underwater nuclear weapon explosion, can be applied to an event like an asteroid impact into the ocean. The formula for H appears to work satisfactorily for a 10400 MT (megaton TNT equivalent) asteroid impact explosion, and the waveform for surface waves is generally similar to that given by Glasstone & Dolan. The R^{-1} attenuation rate adopted in the formula for H appears to be valid. The formula given by Glasstone & Dolan for the period T of the waveform gives a value up to 23% higher than that indicated by SOLA-3D.

References

[1] Glasstone S, Dolan P J, "The Effects of Nuclear Weapons", 3rd edition 1977, Chapter VI, book available online at https://www.fourmilab.ch/etexts/www/effects/

[2] Mader C L, "Numerical modeling of water waves", 2nd edition, CRC Press, 2004

[3] Gardner D J, "Modelling of tsunami waves generated by an underwater landslide", Oct, 2017

[4] "US Underwater Nuclear Testing and Derived Effects Data", http://www.alternatewars.com/BBOW/ABC_Weapons/Nuke_Underwater_Tests_Effects.htm

[5] Crawford D A, Mader C L, "Modeling asteroid impact and tsunami", Science of Tsunami Hazards, Vol 16, No 1, 1998, pp 21-30

[6] Gisler G, Weaver R, Mader C, Gittings M, "Two and three-dimensional simulations of asteroid ocean impacts", Science of Tsunami Hazards, Vol 21, No 2, 2003, pp 119-134