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Development of liquid-structure interaction analysis program for seismic analysis of liquid storage tanks

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1. Introduction

The overall motivation for the study is to investigate whether liquid-structure interaction analysis is necessary for seismic analysis of cylindrical liquid storage tanks. A program for this type of analysis of liquid storage tanks is distributed by NISEE (National Information Service for Earthquake Engineering in the USA) called EXDOMTANK [1].

Limitations of EXDOMTANK include:

(a) The program is limited to the horizontal vibration direction (harmonic 1) for response analysis. It would be desirable to extend its capability to handle the vertical vibration direction as well (harmonic 0).

(b) The program is limited to carrying out time history response analysis. It would be much more convenient to implement a response spectrum analysis capability for the response analysis instead. (In addition, the time history response analysis is limited to zero damping, though it is fairly easy to modify the program to handle the more useful non-zero damping case)

(c) The program is limited to having a maximum of 25 axisymmetric shell elements that can be contact with liquid. It would be desirable to increase this number of elements substantially to provide more detailed response results near the base of the tank.

(d) Another minor limitation is that the program does not consider 'sloshing' of the liquid surface (it assumes the liquid surface is restrained), but it is straightforward to consider this effect in a separate analysis as the tank wall can be regarded as being rigid at the low frequencies associated with sloshing.

The objective is to develop an improved version of EXDOMTANK which addresses limitations (a), (b) and (c).

2. Modifications to EXDOMTANK

2.1 Organisation for improved version of program

EXDOMTANK is supplied by NISEE as four separate programs - SAMMSOR, FLUID, FREQMOD and RESP, which are intended to be run sequentially. The improved version of EXDOMTANK consists of two programs, TANKMODES and TANKRESP.

TANKMODES, which calculates the modes of vibration of a tank and stores relevant data for a subsequent response analysis, incorporates the SAMMSOR, FLUID and FREQMOD programs. TANKRESP, which carries out the response analysis, is a substantially modified version of RESP.

2.2 Capability to handle harmonic zero case

The SAMMSOR and FLUID programs already have an existing capability to handle the harmonic 0 case, but FREQMOD does not. An attempt to calculate vibration modes for harmonic 0 fails in subroutine EIGEN (in FREQMOD) due to zero mass and stiffness being assigned to the tangential degrees of freedom for the axisymmetric shell elements.

Subroutine EIGEN, which determines eigenvalues and eigenvectors for the liquid-structure system, operates on a rectangular matrix which consists of the upper triangle of the system stiffness matrix and the lower triangle of the system mass matrix. The system mass matrix is fully populated, rather than being a diagonal mass matrix as is typically used in most programs distributed by NISEE.

To handle the harmonic 0 case, two new subroutines were introduced - DHZERO and EIGENH0. DHZERO removes all the tangential degrees of freedom from the system stiffness and mass matrices. EIGENH0 is a slightly modified version of EIGEN which works with the modified mass and stiffness matrices produced by DHZERO and assigns zero eigenvector displacements to tangential degrees of freedom when the modeshape data is stored.

It was found that the added mass matrix for the liquid that is calculated in the FLUID program needs to be multiplied by a factor of two for the harmonic 0 case.

2.3 Response spectrum analysis capability

TANKRESP is a substantially modified version of RESP. It retains three slightly modified subroutines from RESP - STRESS, STRAIN and READARY. Subroutines STRESS and STRAIN are associated with calculation of axisymmetric shell element forces and moments at any angle around the circumference, and READARY reads in the banded stiffness matrix for the tank, which is used to calculate base reaction forces and moments for the clamped tank base.

TANKRESP calculates relative displacement and absolute acceleration responses for all nodes at any angle around the circumference, whereas RESP was limited to calculating relative displacement time histories for three selected degrees of freedom.

A new subroutine CALCPF calculates modal participation factors. Further details of this calculation are provided in Section 3.

TANKRESP has been set up as two separate versions, TANKRESP1 and TANKRESP2. TANKRESP1 uses the simpler 'traditional' rules of absolute sum and SRSS (square root sum of the squares) for modal response combination. TANKRESP2 uses more a more sophisticated modal combination procedure which incorporates Rosenbleuth double sum combination and splits the modal response into 'periodic response' and 'rigid response' components as specified in [2]. To implement the more sophisticated procedure, four extra subroutines are employed - RG192A, RG192B, RG192C and RG192D.

2.4 Substantial increase in number of shell elements

The maximum number of axisymmetric shell elements that can be handled has been increased from 30 (with up to 25 elements in contact with liquid) to 200 (with up to 160 elements in contact with liquid). The maximum size of the liquid finite element mesh in the radial and vertical directions has been increased from 25 x 25 to 160 x 160.

The original maximum problem size of 25 elements for the program appears to be modest, but the high bandwidth associated with the liquid-structure interaction analysis method would probably use up a substantial portion of the central memory of a 1970s mainframe computer at this problem size.

Programs distributed by NISEE frequently make use of a large working array which makes increasing the problem size relatively straightforward. EXDOMTANK does not however adopt this practice, requiring the user to have a fairly good understanding of the program in order to increase the maximum problem size. There is less of a requirement to understand the SAMMSOR phase of the program in detail as a separate manual for that program is available which includes instructions on how to increase the maximum problem size.

2.5 Other modifications

(a) Double precision has been implemented.

(b) Numerous changes were introduced to remove CDC mainframe specific coding.

(c) The FLUID program adopts an unusually low g value of 384.0 in/sec^2 in the coding, where g is the acceleration due to gravity. The program was modified to allow the user to enter their own value for g, with a more conventional value of 386.1 in/sec^2 taken as the default. The origin of the unusually low g value in the program probably relates to a slightly low weight density for water of 0.035885 lb/in^3 (or 62.0 lb/ft^3) being adopted for the examples in [1]. A low value of g might then have been adopted to compensate for this slightly low weight density.

3 Calculation of modal participation factors

3.1 Harmonic 1

Adopting the convention in [1], the four degrees of freedom (DOF) per node for the axisymmetric shell element model are in the order u, v, w and dw/dz where u is the vertical DOF, v is the tangential DOF, w is the radial DOF and dw/dz is a rotational DOF.

For harmonic 1, the participation factor for mode i, $(PFi)_{TOT}$, is defined as:

$(PFi)_{TOT} = (PFi)_W - (PFi)_V$

where $(PFi)_W = \{\phi i\}^T [M] \{d\}_W$ and $(PFi)_V = \{\phi i\}^T [M] \{d\}_V$

[M] is the fully populated mass matrix, $\{\phi i\}$ is the modeshape vector, and {d} is a direction vector defined as:

$\{d\}_W = \{0\ 0\ 1\ 0\ 0\ 0\ 1\ 0\ ....0\ 0\ 1\ 0\}^T$

$\{d\}_V = \{0\ 1\ 0\ 0\ 0\ 1\ 0\ 0\ ....0\ 1\ 0\ 0\}^T$

The modeshape vector is 'mass normalised', with $\{\phi i\}^T[M]\{\phi i\} = 1$, and is the full vector, including the degrees of freedom associated with the clamped base node.

3.2 Harmonic 0

For harmonic 0, the participation factor for mode i, $(PFi)_{TOT}$, is defined as:

$(PFi)_{TOT} = (PFi)_U + f_{conv} (PFi)_W$

where $(PFi)_U = \{\phi i\}^T [M] \{d\}_U$ and $(PFi)_W = \{\phi i\}^T [M] \{d\}_W$, and

$\{d\}_U = \{1\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ ....1\ 0\ 0\ 0\}^T$

f_{conv} is a 'conversion factor' which relates the vertical ground acceleration a_v seen by the column of liquid in the tank to an equivalent radial ground acceleration a_r, with $a_r(t) = f_{conv}\ a_v(t)$.

A column of liquid vibrating with an acceleration $a_v(t)$ generates a pressure $\rho_{liq}\ a_v(t)\ z$ on the tank wall where ρ_{liq} is the liquid mass density and z is the depth below the liquid surface.

The total force F_R acting in the radial direction on the cylindrical tank wall due to the liquid pressure associated with a_v is:

$$F_R = \int_0^H \rho_{liq}\ a_v\ z\ 2\pi R\ dz = \pi\ R\ H^2\ \rho_{liq}\ a_v$$

where R is the tank radius and H is the liquid height.

The equivalent radial ground acceleration is then:

$$a_r = \frac{F_R}{M_{radial}} = \frac{\pi R H^2 \rho_{liq} a_v}{M_{radial}}$$

where M_{radial} is the total radial mass, obtained by summing all mass terms associated with the radial direction in the system mass matrix.

This leads to $f_{conv} = \dfrac{\pi R H^2 \rho_{liq}}{M_{radial}}$

4 Test problems for TANKMODES

To test the TANKMODES program, the broad and tall cylindrical steel tanks test problems used in [3] were adopted. Key dimensions of the tanks are shown in Figure 1 below. The symmetry axis is in the vertical direction. The broad tank is also used as one of the example problems in [1].

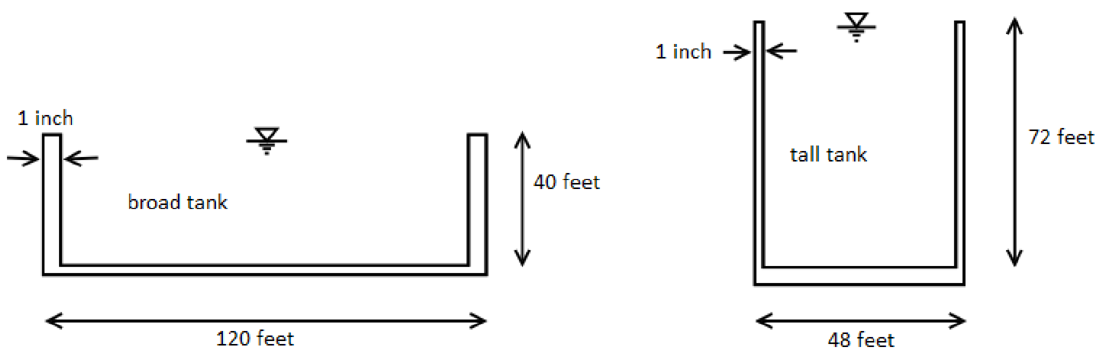

Figure 1 Details of broad and tall tanks used as test problems

The tanks are completely filled with water, and have a uniform wall thickness and no roof structure. The tank bases are clamped.

The steel properties were taken as Young's modulus = 30 x 10^6 psi, mass density = 7.32971 x 10^{-4} lb sec²/in^4 and Poisson's ratio = 0.3.

The water mass density was taken as 9.34505 x 10^{-5} lb sec²/in^4 for the broad tank (to be consistent with [1]), and 9.35690 x 10^{-5} lb sec²/in^4 for the tall tank (which is 1000 kg/m³ converted to pound inch second units). The water is assumed to be incompressible.

5 Comparison of natural frequencies calculated by TANKMODES with literature results

The broad tank test problem was modelled using 20 and 40 shell elements representing the tank wall. For the 20 shell element case, the liquid finite element mesh used was 20 x 20, and for the 40 shell element case 40 x 40 (the n_1 x n_2 mesh refers to n_1 liquid elements in the radial direction and n_2 elements in the vertical direction. The liquid mesh is required to be regularly spaced, which also means the nodes for shell elements representing the tank wall have to be regularly spaced).

Literature results are available in [3] for the horizontal vibration direction (harmonic 1), and [4] for the vertical vibration direction (harmonic 0). The comparison between TANKMODES and literature results for natural frequencies of the first four vibration modes is shown in Tables 1a and 1b below:

Mode	Natural frequency (Hz) for broad tank harmonic 1		
	Literature result	TANKMODES 20 elements	TANKMODES 40 elements
1	6.18	6.20 (+0.3%)	6.19 (+0.2%)
2	11.28	11.41 (+1.2%)	11.31 (+0.3%)
3	15.10	15.54 (+2.9%)	15.20 (+0.7%)
4	17.79	18.72 (+5.2%)	17.99 (+1.1%)

Table 1a Comparison of natural frequencies for broad tank harmonic 1

Mode	Natural frequency (Hz) for broad tank harmonic 0		
	Literature result	TANKMODES 20 elements	TANKMODES 40 elements
1	6.40	6.41 (+0.2%)	6.40 (+0.0%)
2	11.97	12.09 (+1.0%)	11.99 (+0.2%)
3	15.34	15.77 (+2.8%)	15.43 (+0.6%)
4	17.97	18.91 (+5.2%)	18.17 (+1.1%)

Table 1b Comparison of natural frequencies for broad tank harmonic 0

The tall tank was also modelled using 20 and 40 shell elements. The liquid finite element mesh that was adopted was 20 x 20 for the 20 shell element case, and 20 x 40 for the 40 shell element case.

The comparison between TANKMODES and literature results for natural frequencies of the first four vibration modes is shown in Tables 2a and 2b below:

Mode	Natural frequency (Hz) for tall tank harmonic 1		
	Literature result	TANKMODES 20 elements	TANKMODES 40 elements
1	5.31	5.35 (+0.8%)	5.32 (+0.2%)
2	15.64	15.65 (+0.1%)	15.58 (-0.4%)
3	23.24	22.93 (-1.3%)	23.00 (-1.0%)
4	29.85	29.10 (-2.5%)	29.67 (-0.6%)

Table 2a Comparison of natural frequencies for tall tank harmonic 1

Mode	Natural frequency (Hz) for tall tank harmonic 0		
	Literature result	TANKMODES 20 elements	TANKMODES 40 elements
1	6.86	6.82 (-0.6%)	6.81 (-0.7%)
2	18.26	18.06 (-1.1%)	18.16 (-0.6%)
3	26.16	25.62 (-2.1%)	26.09 (-0.3%)
4	31.92	30.91 (-3.2%)	31.96 (+0.1%)

Table 2b Comparison of natural frequencies for tall tank harmonic 0

TANKMODES shows very good agreement with the literature results reported in [3] and [4]. The literature results are actually based on using only twelve shell elements for harmonic 1 and fifteen shell elements for harmonic 0. This suggests that a more accurate approach is being used for modelling the liquid in [3] and [4] than in TANKMODES.

6 Estimation of natural frequencies without using liquid-structure interaction analysis

6.1 Simplified analysis approaches for estimating natural frequency of first mode for tanks in the horizontal vibration direction

6.1.1 Flexural and shear beam formulae

For a thin-walled cylinder, the flexural cantilever beam and shear cantilever beam natural frequencies, f_b and f_s, are:

$$f_b = 0.39569 \frac{R}{H^2} \left(\frac{E}{\rho}\right)^{0.5}$$

$$f_s = \frac{1}{4H\,(4+3\nu)^{0.5}} \left(\frac{E}{\rho}\right)^{0.5}$$

where R is the tank radius, H is the liquid height, E is the Young's modulus of the tank wall, ν is Poisson's ratio for the tank wall, and ρ is an enhanced mass density for the tank wall which takes account of a portion of the liquid mass acting as a non-structural mass. The formula for f_s takes account of the dependence of the shear coefficient on Poisson's ratio.

The combined natural frequency f_{comb} is then given by:

$$\frac{1}{f_{comb}^2} = \frac{1}{f_b^2} + \frac{1}{f_s^2}$$

The portion of the liquid mass that is carried with the tank wall when it vibrates horizontally is known as the 'impulsive mass', and formulae for calculating it are provided in [5].

For a broad tank with D/H $\geq$ 1.3333 (where D is the tank diameter), the impulsive fluid weight W_I relates to the total fluid weight W_T as:

$$\frac{W_I}{W_T} = \frac{\tanh(0.866\ D/H)}{0.866\ D/H}$$

For the broad tank test problem with D/H =3.0, the impulsive mass is about 38% of the total liquid mass.

For a tall tank with D/H < 1.3333, the impulsive fluid weight W_I relates to the total fluid weight W_T as:

$$\frac{W_I}{W_T} = 1.0 - 0.218\ (D/H)$$

For the tall tank test problem with D/H=0.6667, the impulsive mass is about 85% of the total liquid mass.

6.1.2 ASHSD2 model for broad tank

Estimation of the first horizontal natural frequency using beam formulae was expected to be less successful for the broad tank than the tall tank. So another approach that was investigated for the broad tank was to model the tank using another axisymmetric fourier harmonic analysis program distributed by NISEE, ASHSD2, which is described in [6]. ASHSD2 has the advantage of being able to include shell flexibility effects that are not considered in a beam model, but it does not have a specialised liquid-structure interaction analysis capability.

The broad tank was represented using 80 axisymmetric shell elements in ASHSD2, and an enhanced mass density for the tank wall was used as in section 6.1.1.

6.1.3 BNL 52361 procedure for broad tank

A report for seismic design of radioactive waste storage tanks [7] gives a procedure to estimate the first mode natural frequency of a broad tank with a D/H ratio $\geq$ 2.0.

In [7], the first impulsive horizontal mode frequency f_i is given by equation 4-16:

$$f_i = \frac{1}{2\pi}\ \frac{C_i}{H_l} \left(\frac{E t}{\rho t}\right)^{0.5}$$

where H_l is the liquid height, E_t is the Young's modulus of the tank wall and ρ_t is the mass density (not enhanced) of the tank wall. C_i is a coefficient which is determined from Table 4.4 in [7], and is then adjusted using equation 4-18. For the broad tank test problem, C_i is determined by interpolation from Table 4.4.

6.2 Simplified analysis approaches for estimating natural frequency of first mode for tanks in the vertical vibration direction

6.2.1 ASCE 4 water hammer formula

The ASCE 4 standard [5] commentary provides a formula, probably based on water hammer analysis, for estimating the first mode natural frequency f_v of a tank in the vertical vibration direction:

$$f_v = \frac{1}{4H}\left[\rho\left(\frac{D}{tE} + \frac{1}{K}\right)\right]^{-0.5}$$

where H is the liquid height, ρ is the mass density of the liquid, D is the tank diameter, E is the Young's modulus of the tank wall, t is the thickness of the tank wall, and K is the bulk modulus of the liquid.

The bulk modulus K for water was taken as 316,000 psi. The incompressible liquid case, where the 1/K term in the formula is taken as zero, was considered as well as this is more compatible with TANKMODES (which assumes the liquid is incompressible).

6.2.2 BNL 52361 procedure for broad tank

In [7], the first vertical mode frequency f_v is given by equation 4-53:

$$f_v = \frac{1}{2\pi}\,\frac{C_v}{H_l}\left(\frac{E_t}{\rho_t}\right)^{0.5}$$

where H_l is the liquid height, E_t is the Young's modulus of the tank wall and ρ_t is the mass density (not enhanced) of the tank wall. C_v is a coefficient which is determined from Table 4.17 in [7], and is then adjusted using an equation similar to 4-18. For the broad tank test problem, C_v is determined by interpolation from Table 4.17.

6.3 Comparison of natural frequencies

Natural frequencies obtained using the various simplified analysis approaches are compared with TANKMODES results for the natural frequencies of the first mode in horizontal and vertical vibration directions for the test problem tanks in Tables 3 and 4 below.

Tank geometry	Natural frequency (Hz) for first mode harmonic 1			
	TANKMODES 40 element model	flexural and shear beam formulae	ASHSD2 model with enhanced tank wall density	BNL 52361 procedure
Broad	6.19	10.88 (+75.8%)	7.93 (+28.1%)	6.21 (+0.3%)
Tall	5.32	4.91 (-7.7%)	—	—

Table 3 Comparison of TANKMODES and simplified analysis approaches for estimation of first mode natural frequency in the horizontal vibration direction

Tank geometry	Natural frequency (Hz) for first mode harmonic 0			
	TANKMODES 40 element model	ASCE 4 water hammer formula	ASCE 4 formula with incompressible liquid	BNL 52361 procedure
Broad	6.40	7.53 (+17.7%)	7.78 (+21.6%)	6.37 (-0.5%)
Tall	6.81	6.33 (-7.0%)	6.83 (+0.3%)	—

Table 4 Comparison of TANKMODES and simplified analysis approaches for estimation of first mode natural frequency in the vertical vibration direction

Estimating the natural frequency using beam formulae for the horizontal direction and the water hammer formula for the vertical direction is reasonably good for the tall tank, but use of these approaches is less successful for the broad tank. The water hammer formula is possibly more accurate than TANKMODES for the tall tank as it includes the effect of the liquid being compressible.

The ASHSD2 model for the broad tank, which uses shell elements and represents the liquid by an enhanced tank wall density, is less accurate than might have been expected.

The BNL 52361 procedure, which is limited to broad tanks with $D/H \geq 2.0$, shows very good agreement with TANKMODES.

7 Conclusions

1 An improved version of the EXDOMTANK program [1] for seismic analysis of liquid storage tanks has been developed, and is set up as two separate programs, TANKMODES and TANKRESP. The main improvements are (a) a capability to calculate vibration modes in the vertical direction (harmonic 0) has been introduced, (b) the response analysis is now carried out using the more convenient response spectrum analysis method instead of time history analysis, and (c) the maximum problem size that can be handled

has been substantially increased to provide more detailed response results near the base of the tank.

2 The TANKMODES program has been tested, and gives natural frequency results for broad and tall tank geometry cases that are in good agreement with literature results reported in [3] and [4] for the horizontal and vertical vibration directions (harmonic 1 and harmonic 0).

3 It was also checked whether good estimates of the natural frequencies of the first mode in the horizontal and vibration directions can be obtained without using liquid-structure interaction analysis. Use of beam formulae for the horizontal vibration direction and a water hammer formula for the vertical vibration direction provide reasonably good estimates for the tall tank case, but these approaches are less successful for the broad tank case. However very good estimates for both the horizontal and vertical natural frequencies of a broad tank can be obtained using the BNL 52361 procedure [7].

8 References

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